On the construction of band-limited wavelets with the Prouhet-Thue-Morse sequence

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The Prouhet-Thue-Morse sequence has been used in many mathematical fields. For instance, the curious sequence:

\[ \frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{5}{8}, \ldots \]

The limit of this sequence is given by the following theorem:

**Theorem (Robbins)** Let \( \varepsilon_n = (-1)^{t_n} \), where \( \{t_n\}_{n \geq 0} \) is the Prouhet-Thue-Morse sequence. Then

\[
\prod_{n=0}^{\infty} \left( \frac{2n+1}{2n+2} \right)^{\varepsilon_n} = \frac{1}{\sqrt{2}}.
\]

Moreover, the following results are known as applications of the Prouhet-Thue-Morse sequence:

**Theorem (Prouhet)** Define that \( I = \{1 \leq i \leq 2^N; t_i = 0\} \) and \( J = \{1 \leq j \leq 2^N; t_j = 0\} \). The for \( 0 \leq k \leq N - 1 \) we have \( \sum_{i \in I} i^k = \sum_{j \in J} j^k \). In particular when \( N = 4 \),

\[
0^k + 3^k + 5^k + 6^k + 9^k + 10^k + 12^k + 15^k = 1^k + 2^k + 4^k + 7^k + 8^k + 11^k + 13^k + 14^k
\]

for all \( k = 0, 1, 2, 3, 4 \).

**Theorem (Boffa, Point)** Define the Thue-Morse group identities \( I_n \) by :

\( I_0(h, g) \) if \( h = g \); \( I_{n+1}(h, g) \) is \( I_n(hg, gh) \). Then a finite group satisfies a Thue-Morse identity if and only if it is an extension of a nilpotent group by a 2-group.

We investigate the use of invariant cycles to construct band-limited wavelets. We generalize the construction of Lemarié-Meyer wavelets by relaxing the condition on the supports of the low-pass filters. In this talk, we shall show that the Prouhet-Thue-Morse constant has an important connection with the measure of the support of the infinite product of the low-pass filter, and the band-limited wavelets with low-pass filters have larger supports than do the Lemarié-Meyer wavelets.

References


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